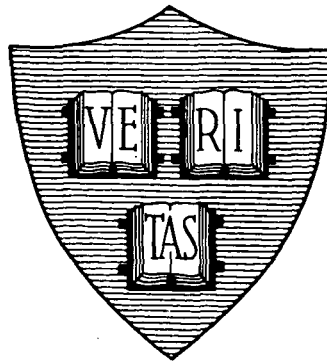


TRACKING VARIATIONS IN THE ALPHA ACTIVITY IN AN ELECTROENCEPHALOGRAM

Technical Report No. 4

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By

K. S. Prabhu

October 1971

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Prepared under Grant NGL 22-007-143

Division of Engineering and Applied Physics

Harvard University • Cambridge, Massachusetts

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ABSTRACT

This report deals with the problem of tracking Alpha voltage variations in an electroencephalogram. This problem is important in encephalographic studies of sleep, effects of different stimuli on the brain, and so on. Very often the Alpha voltage is tracked by passing the EEG signal through a bandpass filter centered at the Alpha frequency, which hopefully will filter out unwanted noise from the Alpha activity. Some alternative digital techniques are suggested and their performance is compared with the standard technique. These digital techniques can be used in an environment where an electroencephalograph is interfaced with a small digital computer via an A/D convertor. They have the advantage that statistical statements about their variability can sometimes be made so that the effect sought can be assessed correctly in the presence of random fluctuations. Also it is possible to track rapid variations in the Alpha activity, which may not be possible in the bandpass techniques due to the limitations imposed by the narrow bandwidth. An attempt is also made to design an optimal filter based on a statistical model and the difficulties which arise are pointed out.

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A. INTRODUCTION

An electroencephalogram (EEG) is a continuous record of the electrical activity of the human brain, as recorded by two or more electrodes connected to the scalp. It has been known for a long time that the time plot of this electrical activity, although unpredictable, often (but not always) exhibits a marked periodicity at a frequency near 10 cycles per second. This cyclic activity is termed the Alpha activity or the Alpha rhythm. A detailed spectral analysis often reveals rhythmic activities at other frequency ranges (β activity, θ activity etc.). However, it is the Alpha activity which has been the subject of much research, and this report concerns itself only with the Alpha activity.

Now, the strength (amplitude) of the Alpha activity varies, even in the same human subject, depending on the subject's neuro-physiological state, presence of external stimuli and so on⁽¹⁾. In particular, detailed studies have been conducted relating the strength of the Alpha activity to different stages of sleep, presence of rhythmic external stimuli at different frequencies etc. Thus an EEG record obtained during a given experiment can be very much a nonstationary phenomenon with the strength of the Alpha activity increasing or decreasing from time to time, slowly or rapidly. There is need for techniques which can effectively track the variations in the Alpha activity.

In the ideal case, the EEG would be an amplitude modulated sinusoid at the Alpha frequency. If this were actually so, the problem of tracking the variations in the Alpha activity would be a simple one - just pass the EEG signal through an envelope detector as shown in Fig. 1.

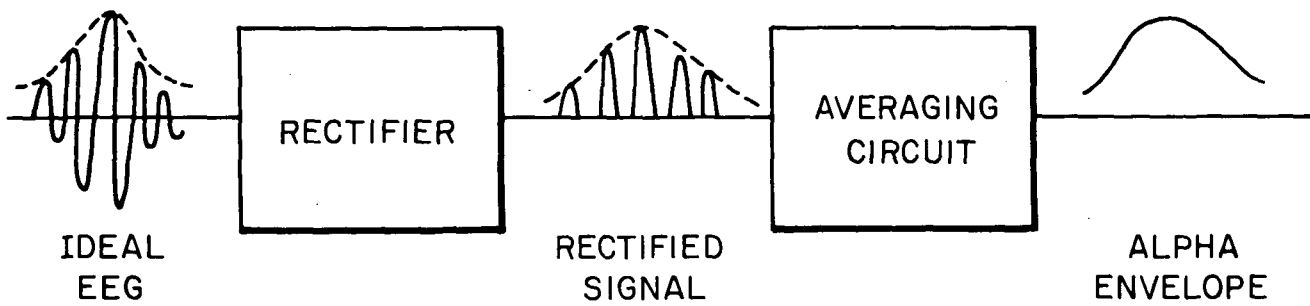


FIG.1 ENVELOPE DETECTOR FOR IDEAL EEG SIGNAL

Unfortunately, the practical situation is complicated by the presence of 'noise' in the EEG signal.

In practice, the 'noise' is sought to be eliminated by passing the EEG signal through a band-pass filter centered at the Alpha frequency. Since the 'noise' is usually at a frequency much higher than the Alpha activity, ordinarily this arrangement works quite satisfactorily. However, Pasquali⁽²⁾ has pointed out that this approach can give rise to serious distortions if the envelope of the Alpha activity has fast variations. What really happens is explained in more detail in Section B.

With the advances in the field of digital computation, it should be possible to track 'fast' variations in the Alpha activity with greater accuracy than is possible with analog band-pass circuits. This report is directed towards the examination of the possibilities with digital techniques. The first two rely on spectral analysis of finite-length time series. The results obtained from these techniques are not very different from those obtained by band-pass filter techniques provided the variations in the Alpha activity are 'slow' enough ('slow' compared to the Alpha frequency itself). For faster variations, the second digital technique shows greater detail, whereas, the band-pass approach produces a smoothing effect, tending to blur the details.

Briefly, the problem with the band-pass techniques is that we need a larger bandwidth to track rapid variations. However, if the bandwidth is made too large, there is the danger that unwanted activity (besides the Alpha activity) may also pass through. With the spectral techniques, we have to analyze shorter lengths of EEG record to track rapid variations. This does not present too many problems because of the availability of

smoothing procedures. In fact, in the literature, spectral analysis has been carried out sometimes on just one second of the EEG record, comprising only ten cycles of the Alpha rhythm.

A third possibility is to use time-domain filtering techniques in which an optimal filter is sought based on some predefined criterion. We would need a statistical model for the EEG record to proceed with this approach. The problem is that any reasonably simple model would have to treat the Alpha frequency as a known parameter. However, in practice, it may be difficult to know it exactly; also the Alpha frequency itself may vary within narrow limits in the course of a given experiment. An attempt to proceed in this direction is outlined in Section E.

B. THE ANALOG METHOD

The analog approach for tracing the fluctuations in the Alpha voltage proceeds as follows. The EEG signal is fed into a bank of narrow band filters which cover the frequency range of interest. The output of each filter would be an amplitude modulated sinusoid at the center frequency of the filter. If the filter were ideal, the highest frequency present in the envelope would be equal to half the bandwidth of the filter. This signal is fed into a square law detector and then into a direct voltage measuring device. If the bandwidth of the filters is small enough, the readings on the voltmeters would not fluctuate too rapidly. These readings will be a measure of the energy contained in each frequency band.

Normally, the Alpha frequency is determined by finding the filter which gives the maximum response during a short 'training'

period. During subsequent tracking of the Alpha voltage, the Alpha frequency is assumed to be constant at this value and only the output of the particular filter is monitored. Perhaps it is possible to take care of Alpha frequency variations by always measuring the output of that filter which gives the maximum output at any given instant. However, this would call for more elaborate circuitry and has not been reported in the literature.

Disadvantages of this Approach.

a. As mentioned above we cannot take into account variations in the Alpha frequency. If we require reasonably stationary readings, the bandwidth of the filters would have to be kept small, which means even small variations in the Alpha frequency are not taken care of.

b. The output depends on the exact transfer function of the filter and the nature of the detecting circuit, which introduces a non-linearity. Thus it is hard to make statistical statements about the variability of the output.

c. The system cannot track rapid fluctuations in the Alpha voltage. This is again because of the small bandwidth of the filters. Suppose the filters are ideal, centered at a frequency f_c with a bandwidth Δf and a signal $A(t) \cos 2\pi f_c t$ is passed through it. If the envelope $A(t)$ contains frequencies greater than $\frac{1}{2} \Delta f$ the filter would not pass them and as a result the envelope would be distorted in passing through the filter. The same effect has been explained by Pasquali⁽²⁾ through the 'inertia' of the filters.

Results: The band-pass filters were simulated digitally and the results obtained are shown in Fig. 2 for comparison with the digital

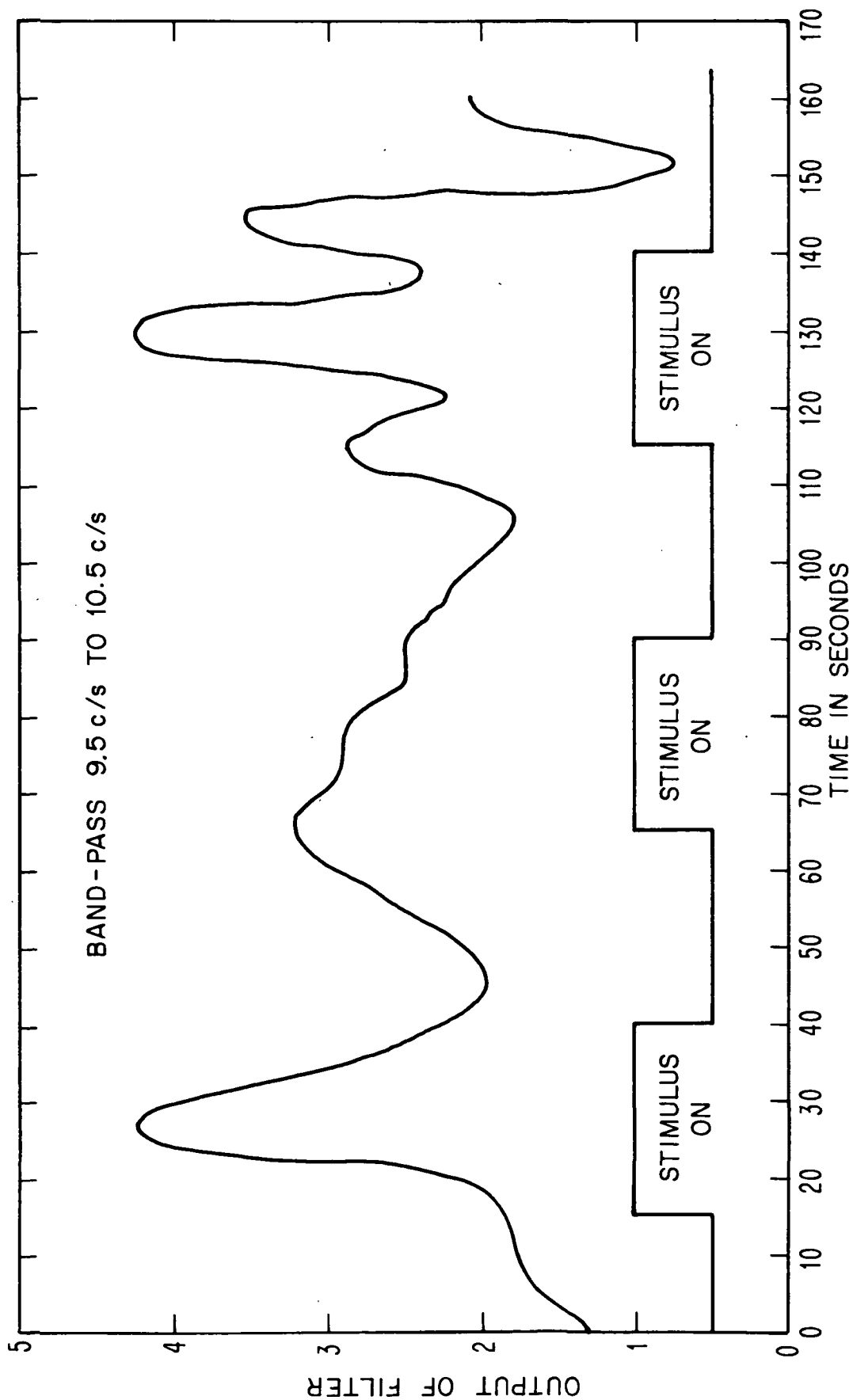


FIG. 2 ALPHA VARIATION AS OBTAINED BY SIMULATION OF BAND-PASS FILTER

techniques described later. A description of the EEG record used appears in the Appendix.

C. DIRECT SPECTRAL COMPUTATION

In this approach, we take a finite length of the EEG record and estimate its power spectrum by any of the available spectral estimation procedures^(3,4). These involve computation of digital autocorrelations, smoothing by a suitable window function and finally Fourier transformation. Several possible window functions and their properties are described in (3). For our purposes, the pertinent details are the following.

Since the objective is to track fast variations in the Alpha voltage, spectral analysis would have to be applied to short lengths of the EEG record in order to avoid smoothing effects. At the same time, we cannot sacrifice resolution (to locate the spectral peak as precisely as possible) and degrees of freedom (a measure of the stability of the spectrum). First of all, we need to be concerned about the sampling rate and the total length of the sampled record. Having obtained the sampled values, we need to know the maximum lag for which the autocorrelations are to be computed. The relationships are as follows:

```

Let      Δt = sampling interval

         n = total number of data points

         m = maximum lag for which autocorrelations are
              computed

```

Then,

$$\text{Resolution} = \frac{1}{2m\Delta t}$$

$$\text{Degrees of freedom} = k \frac{n}{m}$$

(where k is a constant depending on the particular type of smoothing employed).

It is reasonable to require a resolution of 0.5 c/s in order to be able to locate the spectral peak fairly accurately. (In fact, this value has been used frequently in published literature on EEG spectral analysis). This leads to $m\Delta t = 1$ second.

In general, the variability of the spectral estimate goes down as the number of degrees of freedom increases. The exact relationship depends on the particular type of smoothing (lag window) employed. Detailed calculations can be found in references (3) and (4). A practical value for $\frac{n}{m}$ would be 5, which leads to $n\Delta t = 5$ seconds. Consequently five seconds of EEG record would have to be processed in order to be able to locate the spectral peak and compute its height with reasonable accuracy.

It should be noted that the sampling interval Δt did not enter into the calculations so far directly. Actually, it will only determine the highest frequency (Nyquist frequency) of the spectral estimate, which is given by

$$f_N = \frac{1}{2\Delta t}$$

Since we are mainly interested in frequencies around 10 c/s it would be quite adequate to choose $f_N = 50$ c/s giving $\Delta t = 10$ milliseconds and therefore $m = 100$, $n = 500$. So the EEG record will have to be

sampled at 500 points which are 10 milliseconds apart; the lags would then have to be calculated upto a shift of 100 points.

Results and Discussions.

Fig. 3 shows the results of applying this technique to the same piece of EEG record, which was used in Fig. 2. The results are rather similar, as they should be.

As pointed above, we need 5 seconds of EEG record to obtain the required resolution and stability. So this method can track Alpha voltage variations, which take place at about 0.1 cps. To track faster variations, we have to sacrifice either resolution or stability, and there are certainly limitations on how much we can sacrifice on either factor. So it appears that this technique is not very superior to the Analog method insofar as tracking of fast variations is concerned. However, in this technique the EEG signal is processed in such a way that the important work done on the variability of spectral estimates can be applied directly. The parameters can be controlled to keep the variability within limits required by the particular experiment, so we can more easily distinguish the effect sought from the ever-present random fluctuations. This cannot be said of the Analog band-pass technique, which involves nonlinear processing of the EEG signal and hence makes statistical analysis very difficult.

D. SPECTRUM FITTING BASED ON A MODEL

In the previous approach it is clear that the spectral estimate will have to be computed at all frequencies from 0 to f_N (with the

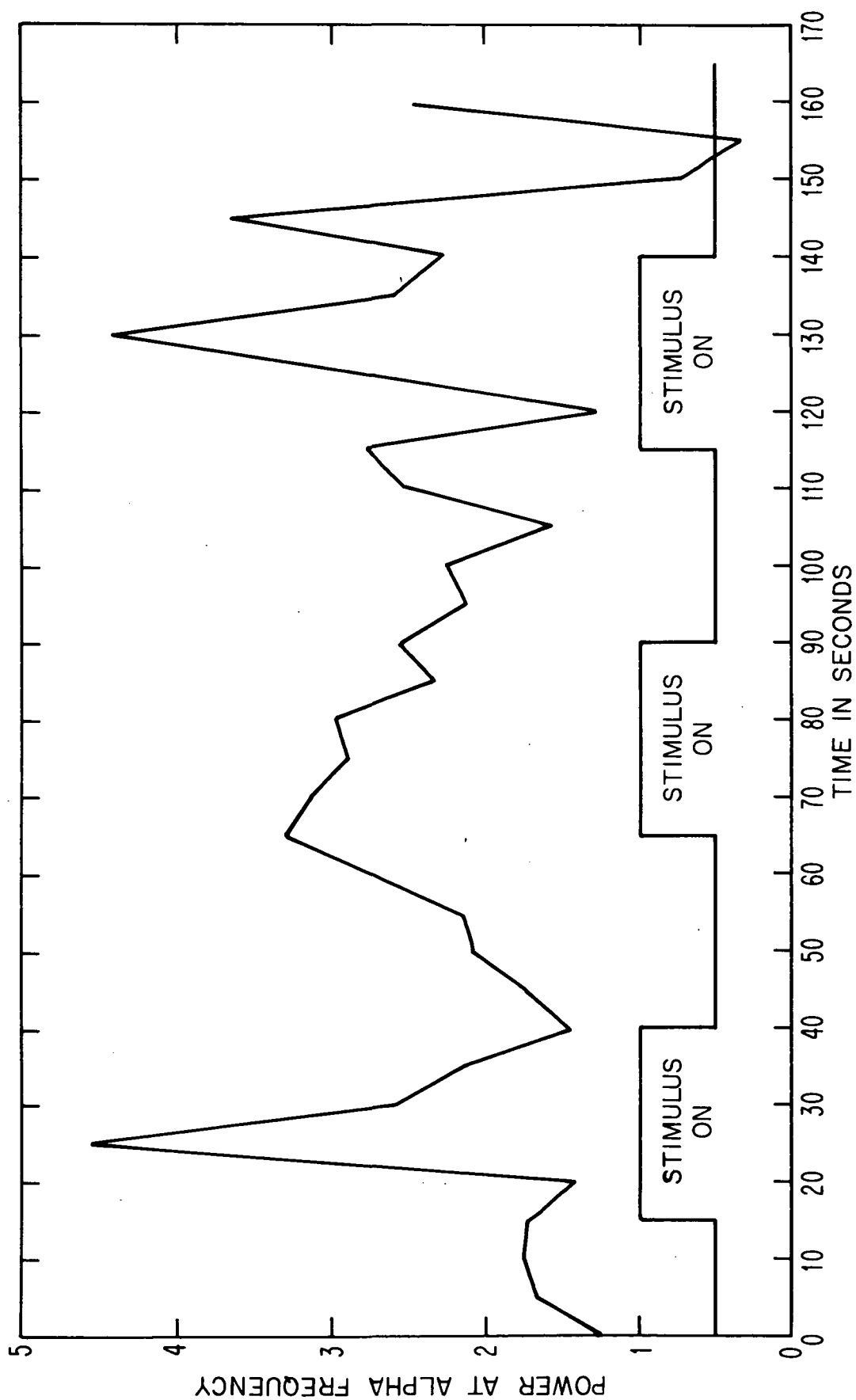


FIG. 3 ALPHA VARIATION AS OBTAINED BY FULL SPECTRAL ANALYSIS
(SPECTRUM SMOOTHED BY TUKEY FILTER)

values chosen in the last section, from 0 to 50 c/s with a resolution of 0.5 c/s). However, to compute the Alpha voltage, only the amplitude at the spectral peak is utilized. Thus it is clear that a lot of useless information has been generated in the process. It should be possible to take advantage of the general shape of the spectrum around the peak and compute the amplitude of the spectral peak to a good approximation. It is shown in this section that this method yields results comparable to those obtained by full spectral analysis.

Now, the simplest way to obtain the kind of spectral peak that EEG records exhibit is to model the EEG record as a second order autoregressive process.

$$x_t - \mu = a_1(x_{t-1} - \mu) + a_2(x_{t-2} - \mu) + z_t \quad (1)$$

where the x_t 's are the equi-spaced sampled values of the EEG, z_t is an uncorrelated zero-mean Gaussian sequence with variance σ_z^2 , μ is the bias and a_1, a_2 are the parameters defining the autoregressive sequence. This has the frequency response function

$$H(f) = \frac{1}{1 - a_1 e^{-j2\pi f \Delta} - a_2 e^{-j4\pi f \Delta}}$$

and therefore the spectrum

$$\Gamma_{xx}(f) = \frac{\Delta \sigma_z^2}{1 + a_1^2 + a_2^2 - 2a_1(1 - a_2) \cos 2\pi f \Delta - 2a_2 \cos 4\pi f \Delta} \quad (2)$$

$$\text{for } -\frac{1}{2\Delta} \leq f \leq \frac{1}{2\Delta}$$

= 0 otherwise, Δ being the sampling interval.

The shape of the spectrum depends on the values of the parameters α_1 and α_2 . By differentiating with respect to f , it is found that a peak or trough occurs at a frequency f_0 given by

$$\cos 2 \pi f_0 \Delta = - \frac{\alpha_1(1-\alpha_2)}{4\alpha_2} \quad (3)$$

$$\text{provided that } |\alpha_1(1-\alpha_2)| \leq |4\alpha_2| \quad (4)$$

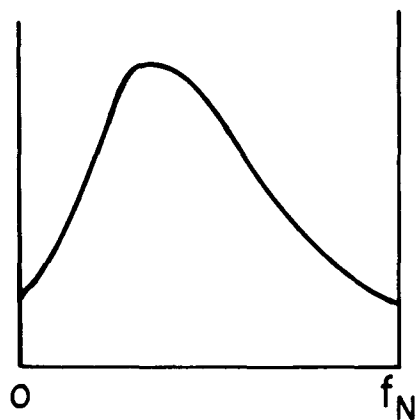
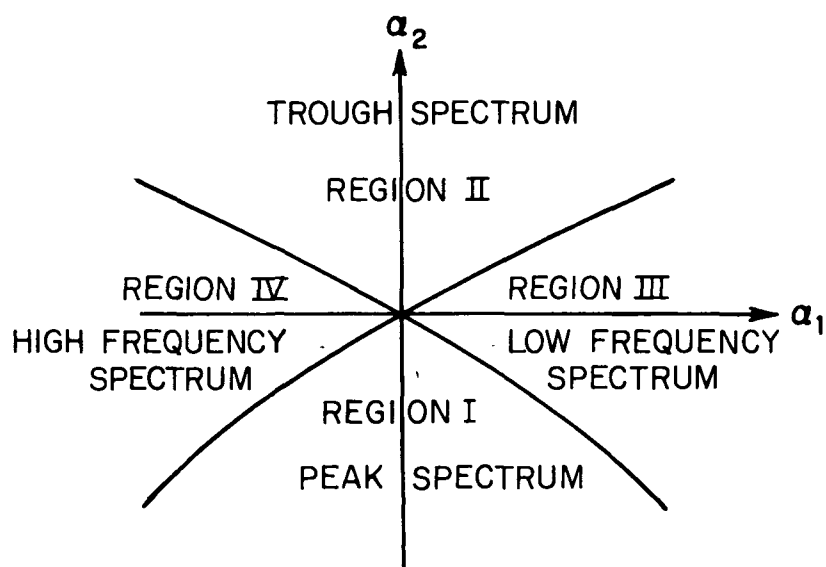
Furthermore it can be shown that the spectrum has a peak or a trough according as $\alpha_2 \lessgtr 0$.

If condition (4) is not satisfied, then we have a low frequency or high frequency spectrum according as $\alpha_1 \lessgtr 0$. The four kinds of spectral behavior and the associated regions in the parameter space (α_1, α_2) are shown in Fig. 4.

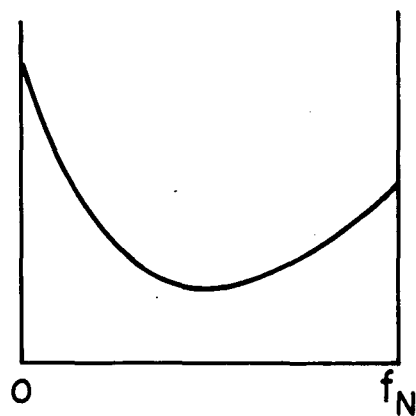
Taking a short length of the EEG record, we can try to fit a second order autoregressive sequence to it and estimate the parameters $\mu, \alpha_1, \alpha_2, \sigma_z^2$. There are well-known least squares procedures to achieve this⁽³⁾. To summarize, let the EEG record be discretised by a time series x_1, x_2, \dots, x_N . Then the best estimates of the parameters are given by the following:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^N x_i$$

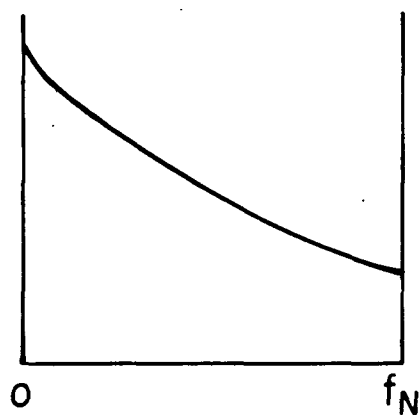
$$\hat{\alpha}_1 = \frac{r_{xx}(1)[1-r_{xx}(2)]}{1-r_{xx}^2(1)}$$



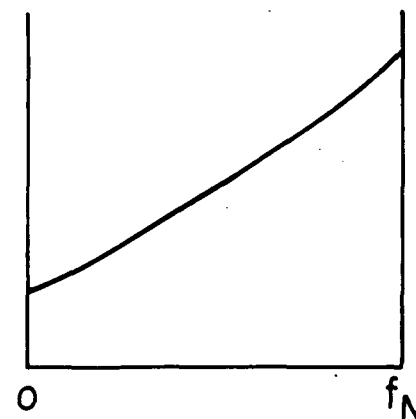
I PEAK SPECTRUM



II TROUGH SPECTRUM



III LOW FREQUENCY SPECTRUM



IV HIGH FREQUENCY SPECTRUM

FIG. 4 KINDS OF SPECTRAL BEHAVIOR OF SECOND ORDER A.R. SEQUENCE.

$$\hat{a}_2 = \frac{r_{xx}(2) - r_{xx}^2(1)}{1 - r_{xx}^2(1)}$$

$$\hat{\sigma}_z^2 = \frac{N-2}{N-5} \{C_{xx}(0) - \hat{a}_1 C_{xx}(1) - \hat{a}_2 C_{xx}(2)\}$$

where

$$C_{xx}(k) = \frac{1}{N} \sum_{i=1}^{N-k} (x_i - \hat{\mu})(x_{i+k} - \hat{\mu})$$

are the autocovariance estimates and,

$$r_{xx}(1) = \frac{C_{xx}(1)}{C_{xx}(0)}$$

and

$$r_{xx}(2) = \frac{C_{xx}(2)}{C_{xx}(0)}$$

are the normalized autocovariance estimates.

Thus the following strategy can be adopted:

1. Sample the EEG record to obtain the time series

x_1, x_2, \dots, x_N :

2. Obtain the estimates $\hat{\mu}, \hat{a}_1, \hat{a}_2, \hat{\sigma}_z^2$

3. From the values of \hat{a}_1, \hat{a}_2 decide in which region of the parameter space the given EEG record falls.

4. If it falls in region II, III or IV, the spectrum does not have a peak in the frequency range of interest; that is, it does not contain any Alpha rhythm.

5. If it falls in region I, compute the peak frequency f_0 using (3); only if it falls within the accepted range of Alpha frequencies, the record need be considered.

6. If f_0 is within the accepted range, compute $\Gamma_{xx}(f_0)$ using (2), which gives the power contained in the Alpha band, from which the Alpha voltage can be deduced.

Results and Discussions.

The length N of the time series is arbitrary (provided it is greater than the number of parameters to be estimated). It determines the variability of the parameter estimates. The sampling interval also can vary within fairly wide limits. Thus the length of the EEG record processed can be made small or large depending on the detail we want to see in the fluctuations of the Alpha activity. Fig. 5 shows the results obtained by processing five seconds of the record just as was done with the previous techniques. The results compare well, showing that the spectrum can be fitted reasonably well with the model of equation (1). Fig. 6 shows part of the EEG record processed in one second lengths. This part was chosen to be one where the magnitude Alpha activity increases rapidly as a result of applied rhythmic stimuli. Fig. 6 shows greater detail than Fig. 5 in this region. Thus, this technique has a distinct advantage in studying rapid fluctuations in the Alpha activity.

E. POSSIBILITIES OF FILTERING IN THE TIME DOMAIN

In this section, the possibility of designing a filter (to track Alpha voltage variations) which is optimal with respect to some criterion defined in the time domain is investigated. First it is necessary to develop a random process model for the EEG record.

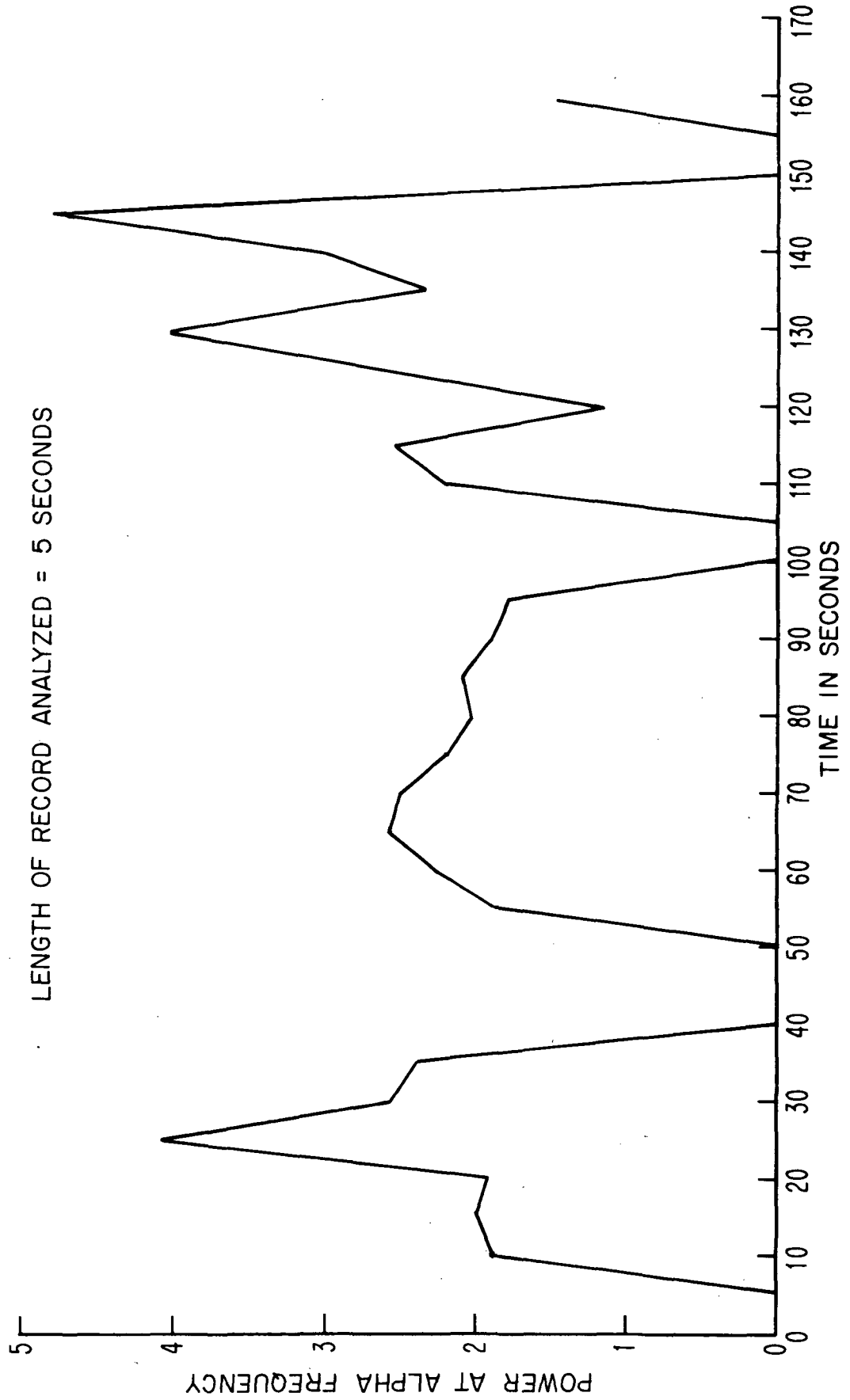


FIG. 5 ALPHA VARIATION AS OBTAINED BY AUTOREGRESSIVE MODEL

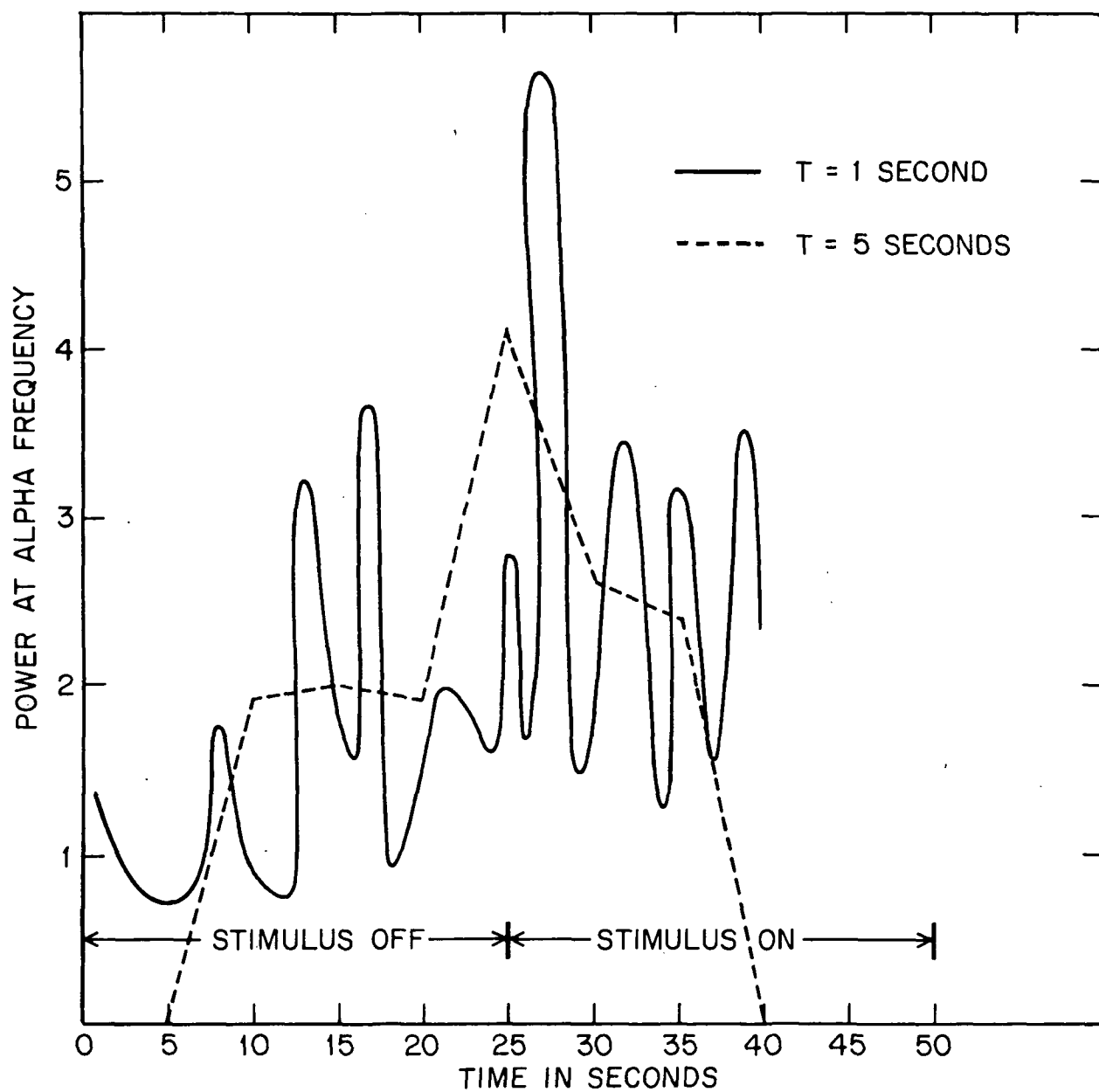


FIG. 6 ALPHA VARIATION OBTAINED BY A.R. MODEL

It may be recalled that in Section D, the EEG record was modeled as a second order autoregressive sequence. This model was found to be good enough to give a rough idea of the spectrum and gave good results with the frequency domain methods. However, for time domain filtering, it is better to have a model which shows the variations of the Alpha activity in a more natural way (rather than through the parameters a_1, a_2).

It was shown in [5] that a plausible random process model for the EEG is obtained by introducing a random perturbation in the phase, this perturbation being a zero-mean Brownian process.

$$x(t) = A(t) \sin [\omega_0 t + \tilde{\theta}(t)]$$

$A(t)$ exhibits the variation in the Alpha activity at frequency ω_0 and $\tilde{\theta}(t)$ is a Brownian process. The problem is to obtain a filtered estimate $A(t)$ from $x(t)$, knowing the Alpha frequency ω_0 and the statistics of the process $\tilde{\theta}(t)$.

Suppose $x(t)$ were discretised with sampling interval Δ

$$x(k\Delta) = A(k\Delta) \sin [\omega k\Delta + \theta(k\Delta)]$$

$$k = 0, 1, 2, \dots, N$$

Knowing the statistics of the sequence $\{\theta(k\Delta)\}$ we can write down the joint probability density of the sequence $\{x(k\Delta)\}$ conditioned on $\{A(k\Delta)\}$, i. e.

$$p[x(0), x(\Delta), \dots, x(N\Delta) | A(0), A(\Delta), \dots, A(N\Delta)]$$

We can try to obtain maximum likelihood estimates of the sequence $\{A(k\Delta)\}$ by maximizing the probability density with respect to $A(0), A(\Delta), \dots, A(N\Delta)$. The mathematical details are omitted; but the

result is that the best estimate of the amplitudes is given by

$$\hat{A}(k\Delta) = \frac{x(k\Delta)}{\sin \omega k\Delta}$$

In other words, the best estimate for the amplitude at any instant is obtained by assuming that the random component of the phase is zero and finding the amplitude which would give the observed value of the EEG signal at the corresponding phase angle.

It is immediately obvious that such filtering would have several drawbacks. First of all it would be very sensitive to any additive noise. Secondly, a precise knowledge of the Alpha frequency is necessary; any errors would tend to cumulate in the phase angle as the filtering proceeds. Experimentally it is known that the Alpha frequency might even change slightly from time to time. It is impossible for any random process model to take care of these changes which occur in an unpredictable fashion. Thus it is evident that any filtering scheme would have to check on the Alpha frequency and the phase angle often.

One possible scheme is to fit a sinusoid to short lengths of the EEG record with the amplitude, the phase angle and the frequency as variable parameters. The problem in the least squares sense would be to minimize.

$$\int_0^T |x(t) - A \cos \omega_0 t - B \sin \omega_0 t|^2 dt$$

where T is the length of the EEG record and A , B , ω_0 are variable parameters. Minimizing with respect to these,

$$A \left(\frac{T}{2} + \frac{\sin 2\omega_0 T}{4\omega_0} \right) + B \left(\frac{T}{2} - \frac{\cos 2\omega_0 T}{4\omega_0} \right) = \int_0^T x(t) \cos \omega_0 t \, dt$$

$$A \left(\frac{T}{2} - \frac{\cos 2\omega_0 T}{4\omega_0} \right) + B \left(\frac{T}{2} + \frac{\sin 2\omega_0 T}{4\omega_0} \right) = \int_0^T x(t) \sin \omega_0 t \, dt$$

and,

$$\begin{aligned} A \int_0^T x(t) t \sin \omega_0 t \, dt - B \int_0^T x(t) t \cos \omega_0 t \, dt \\ + (A^2 - B^2) \left[\frac{T \cos 2\omega_0 T}{2\omega_0^2} - \frac{\sin 2\omega_0 T}{4\omega_0^2} \right] + AB \left[\frac{T \sin 2\omega_0 T}{2\omega_0^2} \right. \\ \left. + \frac{\cos 2\omega_0 T}{4\omega_0^2} - \frac{1}{4\omega_0^2} \right] = 0 \end{aligned}$$

These equations are nonlinear and may have non-unique solutions.

In practice, however, the range of interest of ω is very limited (9 c/s to 11 c/s, say). It is easy to assume different values for ω in this range, solve the first two linear equations in A and B and substitute A, B, ω_0 into the third equation to find the residual. If the portion of the EEG record contained a well-defined Alpha rhythm, it was simple enough to find a value of ω which made the residual nearly zero and thus obtain the least squares fit. If the residual could not be made less than ϵ (some preset value) for any value of ω in the prescribed range, it was assumed that the particular portion of the EEG record contained no Alpha rhythm.

Results.

Here again, the length T of the EEG record processed is arbitrary. Fig. 7 shows the results obtained by processing the same length of the EEG record used in Fig. 6 taking $T = 1$ second to show fast variations. The results are comparable.

F. CONCLUSIONS

A number of digital techniques were suggested to track rapid variations of the Alpha activity in an EEG record. The techniques call for more elaborate equipment than is needed for traditional bandpass technique. However, in an environment which permits their application, the digital techniques can give a better picture of the effect sought and also can track more rapid variations.

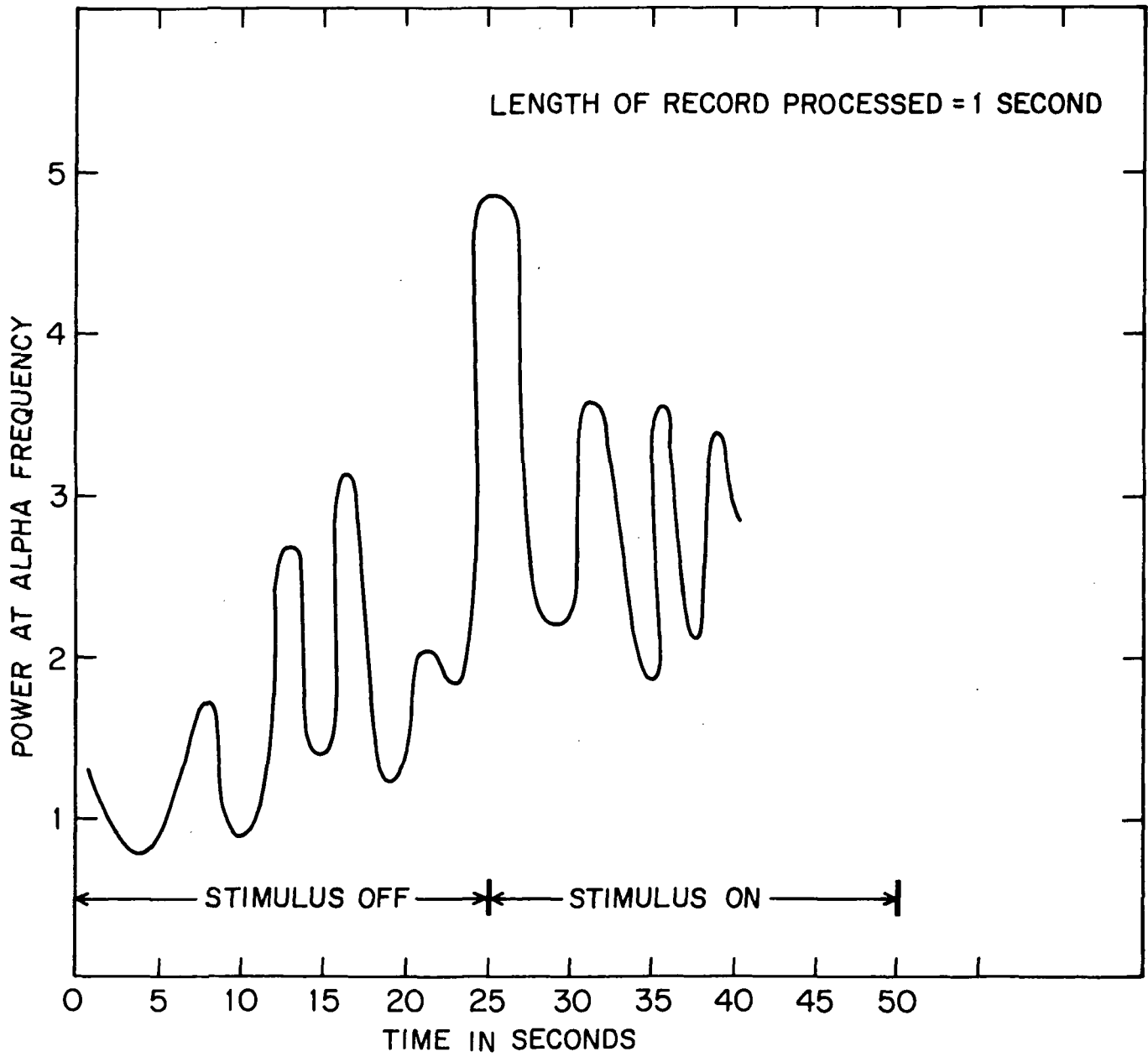


FIG.7 ALPHA VARIATION BY TIME DOMAIN FILTERING

APPENDIX

In this appendix, a description of the EEG data used throughout this report is given.

The EEG record of ten minutes' duration was obtained through NASA-ERC, Cambridge, Massachusetts in 1969. The recording was done in a single sitting from a pair of electrodes located in the left occipital-parietal area. A stroboscope flash turned to the Alpha frequency of the subject was turned on and off regularly. Each on and off period lasted about 25 seconds. It is a well-known fact that rhythmic stimulation at the Alpha frequency increases the strength of the Alpha activity. Thus the EEG signal recorded would be expected to show alternately a rapid increase (with the onset of the flash) and a rapid decrease (when the flash is switched off) in the Alpha activity. The whole record was then digitized at the rate of 100 samples per second.

ACKNOWLEDGMENTS

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